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# Four puzzling paragraphs: Frege on ‘≡’ and ‘=’

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**Abstract:** In §8 of his *Begriffsschrift* (1879), Gottlob Frege discusses issues related to identity. Frege begins his most famous essay, “On Sense and Denotation” (1892), published 13 years later, by criticizing the view advocated in §8. He returns to these issues in the concluding paragraph. Controversies continue over these important passages. We offer an interpretation and discuss some alternatives. We defend that in the *Begriffsschrift*, Frege does not hold that identity is a relation between signs. §8 of the *Begriffsschrift* is motivated by the conflict between two different criteria for sameness of conceptual content of sentences. To resolve that conflict, Frege introduces ‘≡’ in §8 and, thus, circumstances with names as constituents. To the same end, in “On Sense and Denotation,” Frege introduces senses and Thoughts and abandons both ‘≡’ and circumstances. He solves what we call the *Co-instantiation problem*, and disregards, but does not solve, the *Name problem*.

**keywords:** circumstances; conceptual content; denotation; sense; thoughts

## 1 Introduction

In §8 of his *Begriffsschrift* (1879), Gottlob Frege discusses issues related to identity. Frege begins his most famous essay, “On Sense and Denotation” (1892), published 13 years later, by criticizing the view advocated in §8. He returns once more to these issues in the concluding paragraph. Controversies continue over these historically important passages. We offer an interpretation and discuss some alternatives. Starting with the negative points, we argue that

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- (i) In the *Begriffsschrift*, Frege does not hold that identity is a relation between signs; that is, he doesn't hold what is sometimes known as the *Name view* on identity, not even in §8.
- (ii) What Frege introduces in §8 of the *Begriffsschrift*, and represents by '≡,' is not identity but a different relation called 'identity of content,' which presupposes but is different from identity, represented by '='.
- (iii) Neither in the *Begriffsschrift* nor in "On Sense and Denotation," does Frege explain what we call the *Name Problem*: the issue of how we get the information that two signs 'a' and 'b' designate the same thing from a sentence like ' $a = b$ '.

On a more positive vein, we claim that

- (iv) §8 of the *Begriffsschrift* is motivated by the conflict between two different criteria for sameness of conceptual content of sentences: the *inferential criterion* and the *sameness of circumstance criterion*.
- (v) To resolve that conflict, Frege introduces '≡' in §8 and, thus, circumstances with names as constituents.
- (vi) To the same end, in "On Sense and Denotation," Frege introduces senses and Thoughts and abandons both '≡' and circumstances. He solves what we call the *Co-instantiation problem*, and disregards, but does not solve, the *Name problem*.

We proceed by discussing the four critical paragraphs in Sections 1–3: the two paragraphs of §8 of the *Begriffsschrift*, and the opening and closing paragraphs of "On Sense and Denotation." In Section 4, we focus on the *Name problem*, a problem Frege identified but did not quite solve in the *Begriffsschrift*, and that he discarded as a pseudo-problem without argument in "On Sense and Denotation." Finally, in Section 5 we briefly compare our interpretation with some alternatives.

## 2 The two paragraphs of §8 of the *Begriffsschrift*

We'll begin by noting a mistaken view about §8:

[In his *Begriffsschrift*, Frege] adds a special treatment of identity statements to the theory of conceptual content: ' $a = a$ ' has the content that the sign 'a' stands for the same thing as the sign 'a' itself, while ' $a = b$ ' has the content that the sign 'a' stands for the same thing as the sign 'b' (Perry 2012: 141).

This view is suggested by the first paragraph of “On Sense and Denotation,” and is widely accepted by those who write about Frege.<sup>1</sup> But it is wrong.

Frege says *nothing* about the sentences ‘ $a = a$ ’ and ‘ $a = b$ ’ in §8, or anywhere else in the *Begriffsschrift*. The symbol ‘=’ occurs only twice, in an example at the beginning of §1 and in another example, in §5. He also does not, contrary to what he says in “On Sense and Denotation,” maintain in the *Begriffsschrift* that identity is a relation between signs or names, and indeed seems extremely careful not to say that.

## 2.1 Text

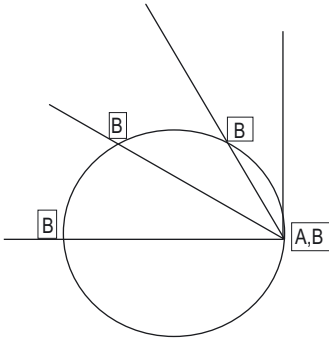
Here is the passage from the *Begriffsschrift*:

### *Identity of Content*

§8. Identity of content differs from conditionality and negation in that it applies to names and not to contents. Whereas in other contexts signs are merely representatives of their content, so that every combination into which they enter expresses only a relation between their respective contents, they suddenly display their own selves when they are combined by means of the sign for identity of content, for it expresses the circumstance that two names have the same content. Hence the introduction of a sign for identity of content necessarily produces a bifurcation in the meaning of all signs: they stand at times for their content, at times for themselves. At first we have the impression that what we are dealing with pertains merely to the *expression* and *not to the thought*, that we do not need different signs at all for the same content and hence no sign whatsoever for identity of content. To show that this is an empty illusion I take the following example from geometry. Assume that on the circumference of a circle there is a fixed point  $A$  about which a ray revolves. When this ray passes through the center of the circle, we call the other point at which it intersects the circle the point  $B$  associated with this position of the ray. The point of intersection, other than  $A$ , of the ray and the circumference will then be called the point  $B$  associated with the position of the ray at any time; this point is such that continuous variations in its position must always correspond to continuous variations in the position of the ray. Hence the name  $B$  refers to something indeterminate so long as the corresponding position of the ray has not been specified. We can now ask: what point is associated with the position of the ray when it is perpendicular to the diameter? The answer will be: the point  $A$ . In this case, therefore, the name  $B$  has the same content as has the name  $A$ ; and yet we could not have used only one name from the beginning, since the justification for that is given only by the answer. One point is determined in two ways: (1) immediately through intuition and (2) as a point  $B$  associated with the ray perpendicular to the diameter (*Begriffsschrift* 20–21).

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<sup>1</sup> A classic formulation can be found in Dummett, “In *Begriffsschrift* Frege held that identity was a relation between names and not between things” (1981a: 544). See also Salmon (1986: 51–54) and Sluga (1980: 151). A more recent formulation can also be found in Corazza and Korta (2015).



**Figure 1:** Geach's diagram to illustrate Frege's example. In Geach and Black (1960).

At the end of §8, Frege introduces a new sign, '≡,' which he calls *identity of content* and gives a definition:

Now let

┆—  $A \equiv B$

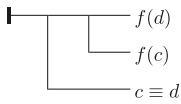
mean that *the sign A and the sign B have the same conceptual content, so that we can everywhere put B for A and conversely* (*Begriffsschrift*, 21).

Before going on with the interpretation, a note on translation. The verb *bedeuten* is in some contexts naturally translated as 'stands for' and other times as 'means' or 'refers.' We use both 'stands for,' 'means' and 'refers' as a translation for '*bedeuten*' as it occurs in the *Begriffsschrift*, and 'denotes' as it occurs in "On Sense and Denotation." This is simply for our convenience; Frege has a somewhat different view of *Bedeutung* in the two works, the result of splitting the concept of *Bedeutung* in the former into *Sinn* and *Bedeutung* in the latter.

## 2.2 Interpretation

In the *Begriffsschrift*, Frege does not discuss sentences with '=', he does not use the identity sign, and he only mentions it twice in two examples, in §1 and §5. He introduces '≡,' which stands for a relation between expressions: *identity of content*. It is the contents that stand in the relation of identity, not the expressions. In the *Begriffsschrift*, the *conceptual contents* of names, predicates and sentences are their referents, that is, objects, concepts, and circumstances. So Frege is using 'identity' in the phrase 'identity of content' in the normal sense, for the relation each object has to itself, and no other. If he didn't mean identity by 'identity,' his explanation of his new symbol would make no sense.

Later in the *Begriffsschrift*, in §20, Frege uses his new symbol to formulate his version of Leibniz's Law:



In more familiar notation:

$$((c \equiv d) \rightarrow (f(c) \rightarrow f(d)))$$

He provides this gloss:

The case in which the content of  $c$  is identical with the content of  $d$  and in which  $f(c)$  is affirmed and  $f(d)$  is denied does not take place. This proposition means that, if  $c \equiv d$ , we could everywhere put  $d$  for  $c$  (*Begriffsschrift* 50).

In this passage, Frege is using the new sign '≡' where logicians would normally use '=', and he would use '=' after the *Begriffsschrift*. But '≡' is *not* a new sign for identity, but a sign for a different relationship, between expressions, identity of content.

This all seems a bit puzzling. Frege does not hold that identity is a relation between names, but that identity of content is, which seems like a reasonable view. He introduces the new symbol '≡' for this relation, and basically retires '='. But he doesn't explain why he retires '='.<sup>2</sup>

## 2.3 Conceptual content and circumstances

A key notion in the *Begriffsschrift* is *conceptual content*. Consider:

I— The Greeks defeated the Persians at Plataea

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<sup>2</sup> He doesn't define '=' either, but this is not too surprising, considering that he only uses it in cases of arithmetical equality. Much later, in the introduction to *Grundgesetze*, Frege details the changes he has made to his formal language since *Begriffsschrift*:

The fundamental signs employed in my *Begriffsschrift* have, with one exception, been used again here. Instead of the three parallel lines I have chosen the ordinary symbol of equality because I convinced myself that it is used in arithmetic to stand for the very thing that I wish to symbolize. In fact, I use the expression 'equal' to stand for the same as 'coinciding or identical with,' and this is just how the sign of equality is actually used in arithmetic to stand for the very thing that I wish to symbolize (*Grundgesetze* 140–141).

This is consistent what he says in the *Begriffsschrift*. His notion of identity is the usual relation of an object with itself, and the notion of identity of content is not needed, he assumes, when talking about arithmetic. See May (2001) and Heck (2003) for an extensive discussion of Frege's view of arithmetical equality.

The vertical bar indicates that we have a judgement; we are asserting that the Greeks defeated the Persians at Plataea. It does not occur, for example, in the antecedent or consequent of a conditional. The horizontal bar is the content stroke; what follows must provide the judgeable content, that is, it must be a sentence. He often calls such contents ‘circumstances’ (*Umstand*), a choice of terminology on which we put some weight. Sentences *bedeuten* circumstances. At this point Frege was not making the distinction between sense (*Sinn*) and denotation (*Bedeutung*), but dealing with a single concept of content.

Conceptual contents and circumstances are important for our interpretation, and we need to emphasize that Frege doesn’t treat ‘circumstance’ (*Umstand*) as a technical term, or even one that needs explanation in the *Begriffsschrift*. But he regularly says that sentences stand for or mean (*bedeuten*) circumstances. A couple of examples:

From §5:

[L]et  $\vdash$ — $A$  stand for (*bedeuten*)  $3 \times 7 = 21$  and  $B$  for the circumstance that the sun is shining ... (*Begriffsschrift* 14).

From §15:

Let  $d$  mean (*bedeuten*) the circumstance that the piston  $K$  of an air pump is moved ...,  $c$  the circumstance that the valve  $H$  is in position  $I$ ,  $b$  the circumstance that ... (*Begriffsschrift* 36).

So, where we write ‘It is a fact that  $S$ ,’ Frege writes

$\vdash$ — $S$

The vertical line is for assertion, and corresponds to ‘it is a fact.’ The horizontal line corresponds to ‘that.’ So

— $S$

identifies the conceptual content, *that S*. The formula,

$\vdash$ — $\neg$ — $S$

corresponds to ‘It is a fact that not- $S$ .’

Following Frege, we use ‘conceptual content’ for a ‘content that can become a judgement,’ (§3), which is the only kind of content that can follow the content stroke, i.e., circumstances. The conceptual content of the whole sentence is determined by the contents of its parts. The content of a name is the object it stands

for. The content of a predicate is a property or relation, the condition that the object or objects named must meet for the sentence to be true.<sup>3</sup>

If a bird flies, the bird *falls under* the concept ( ) *is flying*. Flying is an activity; that is, the concept ( ) *flies* falls under the concept ( ) *is an activity*. So *falling under* is a relation objects have to the properties they instantiate, and properties have to the higher-level properties they instantiate. By 'properties' we mean 'properties and relations.' Circumstances involve objects falling under concepts, i.e., having properties, and concepts falling under higher-level properties, as well as various combinations.

In the terminology we employ, *first-level* circumstances consist of objects having properties and standing in relations. If the objects have the properties and stand in the relations, the circumstance is a fact. Higher-level circumstances consist of concepts falling under concepts. It is a second-level fact, for example, that there are philosophers; that is, that *being a philosopher* falls under *being instantiated*. We call any circumstance a first-level circumstance if it involves objects, even if it also involves properties having properties and standing in relations.

## 2.4 Content problems

Frege tells us that if two sentences have the same conceptual content, they have the same logical consequences (§3). We call this his *inference criterion* for identity of content. As Warren Goldfarb cogently explains in his fine essay "Frege's Conception of Logic" (2010), Frege had a very straightforward view of logic, but one quite different than the view or views of logic that have developed since the works of Tarski and Gödel. Frege thought that logic, like any science, consists of truths, at least in its finished state. It differs from other sciences in being the most general. He thought that the truths of logic were *analytic*, in (more or less) Kant's sense. If we understand them—which may require a lot of work—we will see that they are true, without having to consult what is going on in the world. To see that two sentences have the same content—have the same consequences—one may have to put in a lot of work developing a proof of this fact. But one should not have to go outside one's study to establish some empirical facts.

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<sup>3</sup> Dummett questions the importance of Frege's notion of conceptual content in the *Begriffsschrift*, claiming that it is "plainly a matter which Frege had not at that stage thought through" (Dummett 1981b: 299). We disagree. See Beaney (1996) and Kremer (2010) for a discussion on Frege's notion of content.

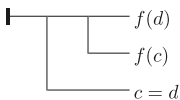
First-level circumstances, especially those involving concrete things like rocks, humans, planets, and words, don't provide contents that meet these criteria. It is a sufficient condition for two sentences to stand for the same circumstance, that they name the same object and predicate the same properties. And, as we will see, there is a conflict between these two criteria for sameness of conceptual content: the inference criterion and the sameness of circumstance criterion.

Consider:

- (1) Hesperus = Hesperus
- (2) Hesperus = Phosphorus
- (3) Hesperus has no moons
- (4) Phosphorus has no moons

(1) and (2) stand for the same circumstance, and so have the same conceptual content, according to the sameness of circumstance criterion. Given that they have the same conceptual content, they should have the same logical consequences by the inference criterion. But (4) follows from (2) and (3), and not from (1) and (3).

Our hypothesis is that it is this problem with conceptual content that led to §8. Frege does *not* tell us that in the *Begriffsschrift*, but, as we shall see, he suggests this motivation in “On Sense and Denotation.” To appreciate the problem we think was bothering Frege, consider a standard formulation of Leibniz's Law using Frege's notation:



Stare at this, keeping in mind that sentences stand for circumstances, and that involving the same objects and properties is a sufficient condition for the identity of circumstances, and hence for identity of conceptual content. Keep the inference criterion for identity of conceptual content firmly in mind; set aside whatever you know about model theory. If you do this, it should seem very puzzling.

The Law *seems* to guarantee that the substitution of ‘*d*’ for ‘*c*’ in ‘*f*(*c*)’ will preserve truth, given that ‘*c* = *d*’ is true. But ‘*c* = *d*’ doesn't seem to tell us anything that justifies the substitution. For ‘*c* = *d*,’ if true, stands for the same circumstance as ‘*c* = *c*’. The two sentences have the same conceptual content. The circumstance they both stand for has a certain object and the relation of identity as constituents. It has no names as constituents. How can this circumstance justify the substitution of names?



There are various ways Frege might have resolved the conflict in criteria. He might have abandoned the inference criterion, but it was very central to his conception of what logic is all about. He might have jettisoned first-level circumstances from his theory, and this is what he eventually does in his theory of sense and denotation. But in the *Begriffsschrift*, he takes another route. He seems to ask what circumstance, instead of  $c = d$ , would make the inference legitimate. His answer is the circumstance that  $c \equiv d$ . This circumstance *does* have names as constituents. Its truth requires that the two names stand for the same thing, so its truth requires that  $c$  and  $d$  are the same thing. But the circumstance that object  $c$  is identical with object  $d$  is not the conceptual content of ' $c \equiv d$ .' And this sentence does not stand for the same circumstance as ' $c \equiv c$ .' The difference in inferential power coincides with, and is explained by, the difference in circumstance.

Given this understanding, Frege is dealing with two connected problems in §8. The first we'll call the *Name Problem*. How do we learn from ' $a = b$ ' that ' $a$ ' and ' $b$ ' stand for the same object? And, why can we infer from ' $a = b$ ' that ' $a$ ' and ' $b$ ' stand for the same object, but not from ' $a = a$ ,' if they both stand for the same circumstance? Frege doesn't formulate the problem, and he doesn't solve it. He simply makes sure that it doesn't arise in *Begriffsschrift*, i.e. Concept-Writing. In Concept-Writing, in the *Begriffsschrift*, we use ' $a \equiv b$ ' which states in a straightforward way what we *seem* to use ' $a = b$ ' to communicate, in spite of its meaning, namely, that ' $a$ ' and ' $b$ ' have the relation of identity of content, that is, stand for, refer to the same thing.

The second problem we'll call the *Co-instantiation Problem*. This is the problem he uses the diagram to explain. We'll use a simpler example. Why would the Babylonians have two names, 'Hesperus' and 'Phosphorus' for the same heavenly body? Because there were two 'ways of determination' associated with them. 'Hesperus' was associated with the property of being the first heavenly body, other than the Sun or the Moon, to appear in the night sky. 'Phosphorus' was associated with the property of being the last heavenly body to disappear from the morning sky, leaving only the Sun and (perhaps) the Moon.

Given this, it seems that one can learn from 'Hesperus = Phosphorus' that the properties of being the first heavenly body to appear at night and the last to disappear in the morning are co-instantiated. But again, this is puzzling; one couldn't learn this from 'Hesperus = Hesperus,' which stands for the same circumstance. But with '≡,' things become clear. 'Hesperus ≡ Phosphorus' tells us that the names stand for the same thing; hence that thing must co-instantiate both ways of determination. 'Hesperus ≡ Hesperus' doesn't tell us this.

Summing up, we think Frege's motivation for §8 comes from a conflict between two criteria for sameness of conceptual content:

**The Inference Criterion:** two sentences have the same conceptual content if and only if they have the same logical consequences,

and

**The Sameness of Circumstance Criterion:** two sentences have the same conceptual content if and only if they stand for the same circumstance.

The conflict stems from the fact that two sentences can stand for the same circumstance and yet have different logical consequences. In particular, different consequences regarding the names and the properties involved. In other words, assuming that ' $a = b$ ' and ' $a = a$ ' both stand for the same circumstance, we get two problems:

**The Name Problem:** From ' $a = b$ ' we can infer that ' $a$ ' and ' $b$ ' stand for the same thing, but not from ' $a = a$ ';

and

**The Co-instantiation Problem:** From ' $a = b$ ' we can infer that  $a$  and  $b$  co-instantiate the same properties, but not from ' $a = a$ '.

### 3 Sense and denotation: first paragraph

Thirteen years after the *Begriffsschrift*, Frege criticized it in the opening paragraph of "On Sense and Denotation." But something a bit odd seems to have been going on. He attributed to himself a view he didn't seem to hold, and then criticized this view, instead of the one he did hold.<sup>4</sup>

#### 3.1 The text and its interpretation

Here is the first part of the first paragraph of "On Sense and Denotation," broken into passages by us, followed by our interpretations:

(A)

Identity gives rise to challenging questions which are not altogether easy to answer. Is it a relation? A relation between objects, or between names or signs of objects? In my *Begriffsschrift* I assumed the latter ("On Sense and Denotation" 209).

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<sup>4</sup> See Perry (2019) for an in-depth discussion of "On Sense and Denotation," and for a presentation of Perry's 'integrated account,' a proposal that integrates Frege's theory in *Begriffsschrift*, with his theory of sense and denotation.

This passage is very puzzling, given what he actually said in the *Begriffsschrift*. He never said that identity was a relation between names or signs. He never held what some people call the *Name view*, *Metalinguistic View* or the *Co-reference Theory of identity*.<sup>5</sup> Our hypothesis is that he is telling us, in a rather opaque way, about the problem that was bothering him when he wrote the *Begriffsschrift* rather than the actual treatment of it that he was led to.<sup>6</sup> A clearer account might have been:

Identity sentences such as ' $a = b$ ' are puzzling. The '=' signifies a relation that holds between an object and itself. But what we seem to learn from such a sentence is that a certain relation holds between the names ' $a$ ' and ' $b$ .'

Instead, the paragraph goes on like this:

(B)

The reasons which seem to favor [the *Name view*] are the following:  $a = a$  and  $a = b$  are obviously statements of differing cognitive value;  $a = a$  holds a priori and, according to Kant, is to be labeled analytic, while statements of the form  $a = b$  often contain very valuable extensions of our knowledge and cannot always be established a priori. The discovery that the rising sun is not new every morning, but always the same, was of very great consequence to astronomy. Even today the identification of a small planet or a comet is not always a matter of course ("On Sense and Denotation" 209).

Here Frege seems to bring up both the Name Problem and the Co-instantiation Problem, without distinguishing them very clearly. In terms of our example, 'Hesperus = Hesperus' is analytic, while 'Hesperus = Phosphorus' is not. That observation seems true, independently of the names being associated with ways of determination—what he will call 'senses.' But the fact that 'Hesperus' and 'Phosphorus' stand for the same thing doesn't seem to qualify as a valuable extension of our knowledge. He seems to be moving seamlessly from the Name Problem to the Co-instantiation Problem.

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<sup>5</sup> See, for example, Thau and Caplan (2001), Heck (2003), Bar-Elli (2006), Dickie (2008), and May (2012).

<sup>6</sup> That Frege's presentation of his view in the *Begriffsschrift* is misleading has also been suggested by Mendelsohn (1982) and Angelelli (1967). On the other hand whether or not Frege was aware of the so-called 'Frege problem' at the time he wrote *Begriffsschrift* is a matter of controversy. Sluga (1980) believes he did, Dummett (1981b) disagrees.

(C)

Now if we were to regard identity as a relation between that which the names ‘*a*’ and ‘*b*’ designate, it would seem that  $a = b$  could not differ from  $a = a$  (i.e., provided  $a = b$  is true). A relation would thereby be expressed of a thing to itself, and indeed one in which each thing stands to itself but to no other thing. What is intended to be said by  $a = b$  seems to be that the signs or names ‘*a*’ and ‘*b*’ designate the same thing, so that those signs themselves would be under discussion; a relation between them would be asserted (“On Sense and Denotation” 209).

In (C), as we interpret it, Frege is telling us more about his motivation for what he said in the *Begriffsschrift* rather than telling us what he actually said there. The last sentence in (C) provide the missing motivation for §8: ‘What is intended to be said by  $a = b$  seems to be that the signs or names ‘*a*’ and ‘*b*’ designate the same thing ...’ In the *Begriffsschrift*, however, this motivated Frege to retire ‘=’ as a symbol, and replace it with ‘≡’, rather than draw the conclusion that with ‘ $a = b$ ’ the signs themselves are under discussion, and ‘=’ asserts a relation between signs.

(D)

But this relation would hold between the names or signs only insofar as they named or designated something. It would be mediated by the connection of each of the two signs with the same designated thing. But this is arbitrary. Nobody can be forbidden to use any arbitrarily producible event or object as a sign for something. In that case the sentence  $a = b$  would no longer be concerned with the subject matter,<sup>7</sup> but only to its mode of designation; we would express no proper knowledge by its means. But in many cases this is just what we want to do. If the sign ‘*a*’ is distinguished from the sign ‘*b*’ only as object (here, by means of its shape), not as sign (i.e., not by the manner in which it designates something), the cognitive value of  $a = a$  becomes essentially equal to that of  $a = b$ , provided  $a = b$  is true. A difference can arise only if the difference between the signs corresponds to a difference in the mode of presentation of that which is designated (“On Sense and Denotation” 209–210).

With the ‘But’ at the beginning of this passage, Frege moves from exposition and apparent justification of his *Begriffsschrift* view, or what he is presenting as his *Begriffsschrift* view, to a critique of the view. After passage (D) Frege moves, still in

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<sup>7</sup> Following Beaney (1997) we translate ‘*betreffen*’ as ‘be concerned with,’ instead of ‘refer,’ as it appears in the translation by Black (Frege (1948)) and Geach and Black (1960).

the same paragraph, to the explanation of his new theory, the theory of sense and denotation.

Frege seems quite right that the relation of identity of content, or co-designation, would hold between names in virtue of their designating something. And, at least as long as we are talking about ordinary proper names, this does seem arbitrary. Just about anything can be used as a name. And it seems that ' $a=b$ ,' or ' $a = b$ ' if we interpret '=' to mean  $\equiv$ , doesn't say anything about the subject matter, and expresses no 'proper knowledge,' in the sense of no knowledge of any but linguistic interest. This is basically the same point he made in §8, with his geometrical example. And it is surely correct that we often do want to say something of interest with a sentence like 'Hesperus = Phosphorus.'<sup>8</sup>

But what comes next is puzzling. Suppose we don't know the ways of determination associated with 'Hesperus' and 'Phosphorus.' Suppose we are told 'Hesperus = Phosphorus' *before* we are told what ways of determination are associated with them. And suppose that, in fact, 'Hesperus = Phosphorus' is true. Why does that mean that the cognitive value of 'Hesperus = Phosphorus' becomes 'essentially equal' to that of 'Hesperus = Hesperus'?

This claim seems wrong. For one thing 'Hesperus = Hesperus' is analytic, on the assumption that 'Hesperus' names something, while 'Hesperus = Phosphorus' is not, even on the assumption that both names name something.

For another, the sentences differ in content according to the inference criterion. As we showed above, (4) follows from (2) and (3) but it doesn't follow from (1) and (3). Suppose now that someone is told (2) 'Hesperus = Phosphorus' and then, that evening and the next morning, has the ways of determination explained to her. She won't learn anything of much interest until she learns the ways of determination, but it seems she will learn something that, once the ways of determination are presented, allows her to make a somewhat astonishing inference, namely, that the first heavenly object to appear in the night sky is the same as the last to disappear from the morning sky. Had she been told (1) 'Hesperus = Hesperus' she would not have been able to make this inference.

### 3.2 The missing solution

It seems that in "On Sense and Denotation," Frege sees his actual *Begriffsschrift* analysis as involving an extra and unnecessary step. He sees the sentence 'Hesperus = Phosphorus' *itself*, not some other sentence involving  $\equiv$ , as carrying,

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<sup>8</sup> See also Perry (2012) and Corazza and Korta (2015).

for semantically competent speakers, the information that the property of being the first planet to appear in the evening and the property of being the last planet to disappear in the morning are co-instantiated.

The ‘ways of determination’ of the *Begriffsschrift* have become senses in “On Sense and Denotation.” So they are part of the content, the Thought expressed. The sentence ‘Hesperus = Phosphorus’ doesn’t express the circumstance that Venus is Venus, but rather expresses the Thought that one thing is both the first planet to appear in the evening and the last planet to disappear in the morning. If we understand the identity sentence (2), we grasp this Thought. There is no need to go through the ‘ $\equiv$ ’ step. We get from ‘Hesperus = Phosphorus’ to a Thought involving significant astronomical information without going through the intermediate step that ‘Hesperus  $\equiv$  Phosphorus.’

But, on this account, what is the solution to the Name Problem? That is, how can one learn that ‘Hesperus’ and ‘Phosphorus’ co-refer from ‘Hesperus = Phosphorus’?

When Frege says in (D),

If the sign ‘*a*’ is distinguished from the sign ‘*b*’ only as object (here, by means of its shape), not as a sign (i.e., not by the manner in which it designates), *the cognitive value of  $a = a$  becomes essentially equal to that of  $a = b$ , provided  $a = b$  is true* (“On Sense and Denotation” 209–210, emphasis added),

one might think he is thinking about the case in which ‘*a*’ and ‘*b*’ are just marks, not yet assigned as names to anything. But then it would make no sense to say, ‘provided  $a = b$  is true.’

In “On Sense and Denotation,” Frege denies that the Name Problem, the difference in what we can learn about *names* from (1) and (2), is a problem after all. He seems to have come to see this as a *pseudo-problem*. But Frege doesn’t tell us *why* this is so.

In fact, Frege’s theory of sense and denotation does not seem to provide a solution to the Name Problem. On the theory of sense and denotation, sentences express Thoughts. Although (1) and (2) denote the same truth-value, they do not express the same Thought. The Thoughts incorporate the senses corresponding to the different names. (1) and (2) differ in content; we learn something of astronomical importance from (2) that we do not learn from (1).

This is a solution to the Co-instantiation problem, but not to the Name Problem. Consider (5):

(5) The first planet to appear in the evening sky is the last planet to disappear from the morning sky.

(5) seems to capture the Thought expressed by (2), since it is the senses of ‘Hesperus’ and ‘Phosphorus,’ not the names, that are contributed to the Thought. But

there is nothing about 'Hesperus' and 'Phosphorus' in (5). Frege's theory provides no route back from sense to name, and so no solution to the Name Problem.

To treat the problem within his theory of sense and denotation, Frege needs to provide us with a Thought that is about names, just as he provided a circumstance involving names in the *Begriffsschrift*. This would require senses of the names themselves, in addition to the senses with which the names are associated. Then he needs to explain how we grasp *that* Thought.

## 4 Sense and denotation: last paragraph

In the last paragraph of 'Sense and Denotation' Frege returns to identity sentences.

### 4.1 Text

Let us return to our starting point!

If we found ' $a = a$ ' and ' $a = b$ ' to have different cognitive values, the explanation is that for the purpose of knowledge, the sense of the sentence, viz., the thought expressed by it, is no less relevant than its denotation, i.e., its truth value. If now  $a = b$ , then indeed the referent of ' $b$ ' is the same as that of ' $a$ ,' and hence the truth value of ' $a = b$ ' is the same as that of ' $a = a$ .' In spite of this, the sense of ' $b$ ' may differ from that of ' $a$ ,' and thereby the sense expressed in ' $a = b$ ' differs from that of ' $a = a$ .' In that case the two sentences do not have the same cognitive value. If we understand by 'judgment' the advance from the thought to its truth value, as in the above paper, we can also say that the judgments are different ("On Sense and Denotation" 230).

### 4.2 Interpretation

Here he treats the Name Problem rather succinctly, with the word 'indeed':

If now  $a = b$ , then *indeed* the referent of ' $b$ ' is the same as that of ' $a$ ,' and hence the truth value of ' $a = b$ ' is the same as that of ' $a = a$ ' ("On Sense and Denotation" 230, emphasis added).

He then notes that the two sentences have different senses, without explaining the 'indeed'; that is, without explaining how we get information about names from a sentence that denotes a truth-value, and expresses a thought that is not about the names, but their denotations—without explaining the Name Problem.

In between the opening and closing paragraphs, Frege has given us an account of quotation:

If words are used in the ordinary way, one intends to speak of their referents. It can also happen, however, that one wishes to talk about the words themselves or their sense. This

happens, for instance, when the words of another are quoted. One's own words then first designate words of the other speaker, and only the latter have their usual referents. We then have signs of signs. In writing, the words are in this case enclosed in quotation marks. Accordingly, a word standing between quotation marks must not be taken as having its ordinary referent ("On Sense and Denotation" 211).

Exceptions [to preservation of truth-value upon substitution] are to be expected when the whole sentence or its part is direct or indirect quotation; for in such cases, as we have seen, the words do not have their customary referents. In direct quotation, a sentence designates another sentence, and in indirect quotation a thought ("On Sense and Denotation" 218).

Does this provide an explanation to the Name Problem? It doesn't seem so. The issue is how we learn from

(2) Hesperus = Phosphorus

that 'Hesperus' and 'Phosphorus' stand for the same object, not how we learn from

(2') 'Hesperus = Phosphorus'

or

(2'') 'Hesperus = Phosphorus' is true

that they do. The last two sentences express what we seem to learn when we read (2), but it doesn't explain how we learn it.

It seems most plausible that Frege still thinks that the Name Problem is a pseudo-problem, rather than thinking it is a real problem he has solved by the end of "On Sense and Denotation."

## 5 Is the name problem a pseudo-problem?

Suppose a bright, but philosophically gifted, student stares at a formulation of Leibniz's Law such as,

$$(a = b) \rightarrow (f(a) \rightarrow f(b))$$

and find it mysterious, in the way we imagined Frege to have found it mysterious, and encouraged the reader to find it mysterious. 'If the the antecedent is true,' she says to her instructor, 'it puts the same requirements, on the same object(s), as  $'a = a'$ .' But we couldn't draw the inference in that case. The antecedent seems to tell us that ' $a$ ' and ' $b$ ' co-refer—but isn't that a 'use-mention' confusion? How does the identity of object(s) tell us anything that permits substituting the names? The instructor might reply, 'Look at it from the model-theoretic point of view. Any model in which  $'a = b'$  and  $'f(a)'$  is true will be one in which  $'f(b)'$  is true. You don't get that result with  $'a = a'$ .'



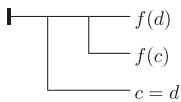
The instructor might be impressed with the student's bafflement. But she regards it as a pseudo-problem. It evaporates as soon as one looks at the truth-conditions of the sentence from another angle, not in terms of what it says, but what the semantics of the language requires for a sentence like it to be true.

From what 'Hesperus = Phosphorus' says, one can learn that Hesperus and Phosphorus are the same planet, but not that the names used refer to the same thing. On Frege's view, the sentence expresses a Thought; something like, *There is a planet that is both the first planet to appear in the night sky and the last to disappear from the morning sky.* On a direct reference view, it expresses the singular proposition that Venus is Venus. On neither view does the truth of what it expresses put any conditions on the names 'Hesperus' and 'Phosphorus.'

If we don't worry about what the sentence says, but simply the conditions of truth imposed by its semantics, we see that the sentence is true in all models in which '=' is assigned a relation that holds between the referents of the two names. If we assume that '=' is a logical term, so is in effect assigned to identity in all models, we learn that in all models in which the sentence is true, 'Hesperus' and 'Phosphorus' are assigned to the same object. If the sentence is true, the names co-designate.

We agree with the later Frege that this was a pseudo-problem for the earlier Frege, that he could ignore, and the earlier Frege should have ignored. But the later Frege doesn't say *why* it is a pseudo-problem. Since he had no opportunity to study Tarski and Gödel one can hardly blame him.

Still, we think the earlier Frege has the equipment for such a treatment almost in his grasp with his concept of bifurcation, and his clear if not explicit semantics in terms of conceptual content. Suppose Frege wrote the *Begriffsschrift* without §8 and without replacing '=' with '≐.' He found one of his students staring at the version of Leibniz's Law in this hypothetical version:



When asked what's bothering her, the student says, 'if ' $c = d$ ' is true, it stands for the same circumstance as ' $c = c$ .' So if we replace the first by the second, since they have the same conceptual content, the same things should follow. But surely, given the replacement it wouldn't follow that  $f(d)$ .

At that point, it seems to us, Frege could have said something like this:

You have to keep in mind how my theory of the truth-conditions of sentences works. On that theory, a sentence of the form ' $cRd$ ' is true iff ' $c$ ' stands for an object, ' $d$ ' stands for an object, and the two objects stand in the relation meant by ' $R$ '. Since ' $R$ ' means identity, the sentence

' $c = d$ ' is true iff ' $c$ ' and ' $d$ ' stand for the same object. This assures us that replacing ' $c$ ' with ' $d$ ' won't affect the truth-value. But it doesn't work with ' $c = c$ '.

You must keep in mind that there is always a bifurcation in our use of language. Sometimes we are happy to grasp what the sentence says, the circumstance it stands for, the conditions its truth imposes on the properties and objects the expressions stand for. At other times, we need to reflect on *how* it says what it says; that is, what conditions its truth imposes on the expressions used, simply in virtue of the structure of the sentence and the way the language works.

If the student then says, 'Does that mean you are changing your conception of logic?', we aren't sure what Frege should say.

## 6 Conclusion

The first paragraph of "On Sense and Denotation" is probably the single most-read paragraph in analytic philosophy. It is remarkable how misleading it is. The author attributes himself a view on identity that never held, the Name View, and proceeds to criticize it. The standard interpretation takes the paragraph at face value and attributes Frege the Name View, first, and the Object View later on. Further, the standard interpretation does not distinguish between two different relations that are present in the *Begriffsschrift*, identity ( $=$ ) and identity of content ( $\equiv$ ).

On a recent set of papers, Thau and Caplan (2001), Heck (2003), and Bar-Elli (2006)<sup>9</sup> have had an interesting discussion about the standard interpretation, its vices and virtues. Without getting into much detail, we think our proposal sheds light on some critical points of the discussion.

First, it is surprising that not many commentators pay attention to the difference between  $=$  and  $\equiv$ , and very few take them to be *two different relations*.<sup>10</sup> Both Thau and Caplan and Heck, for instance, accept the common view in this regard, and take ' $=$ ' and ' $\equiv$ ' to be notational variants of the same relation—identity. But this is quite surprising, why would Frege introduce a new symbol for identity? Why have two symbols for the same relation in the *Begriffsschrift*? He used two symbols, so it seems reasonable to conclude that he thought they had two different meanings. To repeat, ' $=$ ' and ' $\equiv$ ' stand for different relations. Frege never held the Name View about identity in the *Begriffsschrift*, or anywhere else

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<sup>9</sup> See also Dickie (2008).

<sup>10</sup> This might have been caused by Frege's own confusing remarks in "On Sense and Denotation," as we have discussed. Also, most of the key literature on Frege, and in particular Dummett (1981a, 1981b), accord with this. For exceptions, see Mendelsohn (1982), May (2001, 2012), and Bar-Elli (2006).

for that matter. He held that *identity of content* is a relation between names, but not that *identity* is.

Second, the Name View of identity, which is misattributed to the *Begriffsschrift* by many—including, perhaps, Frege himself in “On Sense and Denotation” —, is often called the also *Metalinguistic View*. These terms are used quite differently by Thau and Caplan (2001). For these authors,

On the first, metalinguistic view—which we'll call the *name view*—‘ $\alpha = \beta$ ’ expresses the thought that the *names*  $\alpha$  and  $\beta$  have the same referent; whereas, on the second view, non-metalinguistic view—which we'll call the *object view*—‘ $\alpha = \beta$ ’ expresses the thought that  $a$  and  $b$  are the same object, where  $a$  and  $b$  are the objects named by  $\alpha$  and  $\beta$  (Thau and Caplan 2001: 161).

Notice that the distinction here is not between two views about identity but between two views about the meaning of identity statements or, rather, as they put it, about the kind of thoughts expressed by identity statements. They argue that Frege holds the metalinguistic view on the meaning of identity statements in *Begriffsschrift* and that he *never abandons* it.

We think that framing the discussion like that is misleading. First, because there is no metalinguistic view about *identity* statements in *Begriffsschrift*, there is a *Name View* of *identity-of-content* statements. But, second, and even focusing on the latter, what an identity-of-content statement says is: ‘ $\alpha \equiv \beta$ ’ stand for the circumstance where the *names*  $\alpha$  and  $\beta$  have the same content. ‘Have the same content’ and ‘have the same referent/denotation’ might be practically synonyms, but substituting ‘expressing a thought’ for ‘stand for a circumstance’, as Thau and Caplan seem to do, is far from innocuous, especially when talking about the similarities and differences between *Begriffsschrift* and “On Sense and Denotation.” In the former, first-level circumstances contain objects and relations, so the circumstances that identity-of-content statements stand for contain two names and a relation. In the latter, the thoughts expressed by identity statements contain senses, and not names or relations.

Contrary to what Thau and Caplan claim, what Frege holds in *Begriffsschrift*, and never rejects in “On Sense and Denotation,” is the view that if ‘ $a \equiv b$ ’ is true and, therefore, ‘ $a = b$ ’ is also true, then the referent of ‘ $b$ ’ is the same as the referent of ‘ $a$ .’ He says as much in the last paragraph of “On Sense and Denotation.” But this view is neither the Name View of identity, as we understand it, nor the metalinguistic view of identity statements, as Thau and Caplan (2001) understand it.

According to our interpretation, Frege found it natural at this point in his thinking to take sentences as standing for or meaning circumstances, in pretty much the sense the phrase ‘states of affairs’ is now commonly used. We are taking the word seriously, perhaps more seriously than Frege himself did, but it is helpful

in understanding the problems Frege had in the *Begriffsschrift* with identity, problems which he thought he solved later with his theory of sense and reference.

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## References

- Angelelli, Ignacio. 1967. *Studies on Gottlob Frege and traditional philosophy*. Dordrecht: D. Reidel.
- Bar-Elli, Gilead. 2006. Identity in Frege’s *Begriffsschrift*: Where both Thau-Caplan and Heck are wrong. *Canadian Journal of Philosophy* 36(3). 355–370.
- Beaney, Michael. 1996. *Frege: Making sense*. London: Duckworth.
- Beaney, Michael (ed.). 1997. *The Frege reader*. Oxford: Blackwell.
- Corazza, Eros & Kepa Korta. 2015. Frege on subject matter and identity statements. *Analysis* 75(4). 562–565.
- Dickie, Imogen. 2008. Informative identities in the *Begriffsschrift* and “on sense and denotation”. *Canadian Journal of Philosophy* 38(2). 269–288.
- Dummett, Michael A. E. 1981a. *Frege: Philosophy of language*, 2nd edn. Cambridge, MA: Harvard University Press.
- Dummett, Michael A. E. 1981b. *The interpretation of Frege’s philosophy*. Cambridge, MA: Harvard University Press.
- Frege, Gottlob. 1948 [1892]. Sense and reference, Max Black (trans.). *The Philosophical Review* 57. 207–230.
- Frege, Gottlob. 1967 [1879]. *Begriffsschrift*: A formula language, modeled upon that of arithmetic, for pure thought, Stefan Bauer-Mengelberg (trans.). In Jean van Heijenoort (ed.), *From Frege to Gödel: A source book in mathematical logic, 1879–1931*. Cambridge, MA: Harvard University Press.
- Peter Geach & Max Black (eds.). 1960. *Translations from the philosophical writings of Gottlob Frege*, 2nd edn. Oxford: Basil Blackwell.
- Goldfarb, Warren. 2010. Frege’s conception of logic. In Tom Ricketts (ed.), *Cambridge companion to Frege*, 63–86. Cambridge: Cambridge University Press.
- Heck, Richard. 2003. Frege on identity and identity-statements: A reply to Thau and Caplan. *Canadian Journal of Philosophy* 33(1). 83–102.

- Kremer, Michael. 2010. Sense and reference: The origins and development of the distinction. In Tom Ricketts (ed.), *Cambridge companion to Frege*, 220–293. Cambridge: Cambridge University Press.
- May, Robert. 2001. Frege on identity statements. In Carlo Cecchetto, Gennaro Chierchia & Maria Teresa Guasti (eds.), *Semantic interfaces: Reference, anaphora, and aspect*, 1–62. Stanford: CSLI.
- May, Robert. 2012. What Frege's theory of identity is not. *Thought: A Journal of Philosophy* 1(1). 41–48.
- Mendelsohn, Richard L. 1982. Frege's *Begriffsschrift* theory of identity. *Journal of the History of Philosophy* 20(3). 279–299.
- Perry, John. 2012. *Reference and reflexivity*, 2nd edn. Stanford: CSLI.
- Perry, John. 2019. *Frege's detour*. Oxford: Oxford University Press.
- Salmon, Nathan. 1986. *Frege's puzzle*. Cambridge, MA: Harvard University Press.
- Sluga, Hans. 1980. *Gottlob Frege*. Boston: Routledge & Kegan Paul.
- Thau, Michael & Ben Caplan. 2001. What is puzzling Gottlob Frege? *Canadian Journal of Philosophy* 31(2). 159–200.